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Synchronization of Nonidentical Phase Oscillators in Directed Networks

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The synchronization behavior of Kuramoto-type nonidentical oscillators on directed networks is numerically studied. We start from the pure structure of the directed tree and build directed networks by splitting each edge at a given probability p with the total incoming edge weight of each vertex preserved. As more edges are split for a given network size, the synchronization order parameter is found to decrease and then to increase beyond the value for the original tree structure at $p = 0$. We also investigate the synchronization behaviors of networks in the thermodynamic limit via an analytic approach within the linear approximation and a finite-size scaling numerical analysis: Whereas nonidentical oscillators cannot be synchronized at any finite coupling strength in the infinite-sized tree networks at $p = 0$, splittings of sufficient number of edges are shown to lead to a phase transition at a finite coupling strength. From these findings, we discuss the role of bidirectional information transfer in the synchronization behavior of directed networks.

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I. INTRODUCTION

The synchronization behavior of complex networks has drawn intensive research interest because abundant examples exist in biological, sociological and physical systems [1–3]. The star network in which one vertex is connected to every other vertex is, of course, a trivial example of an optimized structure for synchronization. Recently, beyond such a trivial example far from real networks, it has been revealed, via the master stability function (MSF) approach [1,4] applied for identical oscillators, that there exist a set of directed networks embedding the tree structure a perfect synchronizability [5, 6].

The MSF analysis makes a linear perturbation to the fully synchronized state and checks the stability within the linear response region. If the synchronous state is stable at a given coupling strength, the perturbation exponentially decays, otherwise, the system approaches an asynchronous state as time goes on. The practical beauty of the MSF is that one does not need to solve the equations of motion for oscillators; the structure of network connections, *i.e.*, the adjacency matrix, together with the parameters in the equations of motion, is sufficient to yield a result in a MSF analysis. In comparison, other works exist in which *nonidentical* oscillators are assumed, with the celebrated Kuramoto model [7,8] being the best-known example. In a system of coupled Kuramoto-type phase oscillators, it is impossible for oscillators to be completely synchronized at any finite coupling strength due to the different intrinsic frequencies and the research focus has been put on the critical point at which *partial synchronization* begins to develop. In short, in the research literature, there have been two different, but closely related, ways to study synchronization: stability study of fully synchronized identical oscillators and study of the onset of partial synchronization of nonidentical oscillators.

In the present paper, we take the latter approach and investigate the synchronization of *nonidentical* oscillators put on directed networks that embed trees and we compare the results with those for *identical* oscillators [5]. The synchronization behavior in tree networks with oscillators only at the bottom level has been investigated [8, 9]. In the present study, in contrast, we put the Kuramoto-type phase oscillators at all vertices in the tree network and aim to study systematically how the synchronization behavior changes with the changing network structure. Via an extensive numerical study of nonidentical phase oscillators, we reach the conclusion

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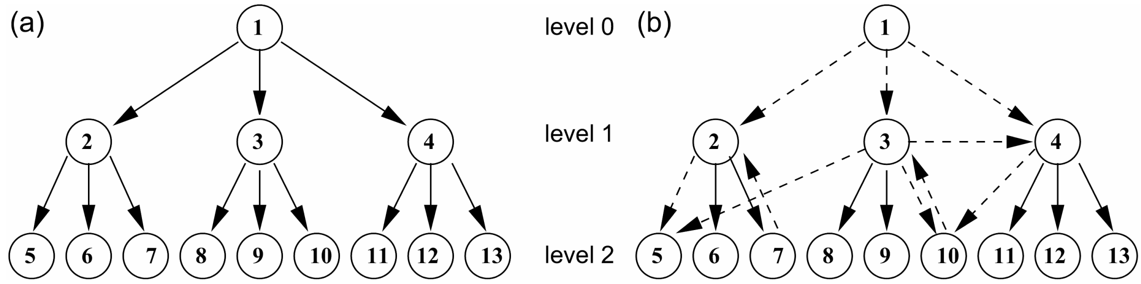

Fig. 1. (a) An initial directed tree network with the completely hierarchical structure [5,6] is first built with the branching ratio 3 and the number of layers from the top vertex $L = 3$. The total number of vertices is written as $N = (3^L - 1)/2 = 13$. (b) Each vertex, except for the top vertex marked as 1, is visited and at a given splitting probability p, the weight of the incoming edge is reduced by half; then, a new incoming edge of weight 1/2 is made from another randomly chosen vertex. Differently from the original structure in (a), there now exist edges destroying the hierarchical structure, *e.g.*, $7 \to 2$ and directed loops are formed as well, *e.g.*, $2 \to 7 \to 2$ and $3 \to 4 \to 10 \to 3$. A solid (dotted) arrow indicates that its edge weight is unity (one half).

that a more abundant bidirectional information transfer can, indeed, yield a bigger scale of collective synchronization behavior within our network model. It is to be noted that our network model does not belong to the nonhierarchical networks optimizing the synchronization of identical oscillators [10]. We also investigate the thermodynamic properties. In the infinite tree network, we show analytically within the linear approximation that there exists no synchronous state at any finite coupling strength. In contrast, as sufficient numbers of incoming edges are split, we find via numerical simulations and a finite-size scaling analysis that a synchronization phase transition develops at a finite coupling strength, analogous to the result of a previous study of phase oscillators in small-world networks [11].

The present paper is organized as follows: In Sec. II, we describe the construction of directed networks and nonidentical phase oscillators. Numerical results and a simple analytic calculation are presented in Sec. III. Finally, we summarize our work in Sec. IV.

II. DIRECTED NETWORKS AND PHASE OSCILLATORS

We construct directed networks as shown in Figure 1. First, a directed tree network with L layers and the branching ratio b is built with the number of vertices being $N = (b^L - 1)/(b - 1)$ [see Figure 1 (a)]. Initially, each vertex other than the one at the top has one incoming edge whose weight is unity, satisfying the conditions for optimized synchronizability for identical oscillators [5, 6]. Visiting every vertex, except for the top vertex, we reduce the weight of the incoming edge to 1/2 and add one incoming edge of weight 1/2 from another randomly chosen vertex at the splitting probability p. The network is a pure tree for $p = 0$, whereas every vertex, except for the top vertex, has two incoming edges of half weights when $p = 1$. It is to be noted that the above edge-splitting procedure preserves the normalized input strength of each vertex. In our network model, the resulting networks at $p > 0$ embed the tree structure with additional links, which generate directed loops, *e.g.*, 2→7 and 3→4→ 10→3 in Figure 1, violating the perfect synchronizability conditions for identical oscillators [5,6].

In the Kuramoto model with N coupled oscillators, the equations of motion are given by

$$\dot{\theta}_i = \Omega_i + K \sum_{j=1}^{N} W_{ij} \sin(\theta_j - \theta_i), \tag{1}$$

where θ_i is the phase of the ith oscillator and Ω_i is the uncorrelated quenched intrinsic frequency generated from the Gaussian distribution with the null average ($\langle \Omega_i \rangle = 0$ with the ensemble average $\langle \cdots \rangle$) and the unit variance ($\langle \Omega_i \Omega_j \rangle = \delta_{ij}$). In the limiting case of a vanishing variance, which corresponds to identical oscillators and can be approached by $K \to \infty$ at a unit variance by rescaling of the time variable, the system is completely synchronized regardless of the splitting probability. The element W_{ij} of the $N \times N$ weighted adjacency matrix is simply the weight of the directed edge from j to i; thus, $W_{ij} = 0$ if no element connects j to i.

The synchronization order parameter $\Delta(t)$ as a function of the time t is defined as [7]

$$\Delta(t) = \frac{1}{N} \left\langle \left| \sum_{i=1}^{N} e^{i\theta_i(t)} \right| \right\rangle. \tag{2}$$

In addition, the time-averaged value $\bar{\Delta}$ of $\Delta(t)$ is measured after steady state is achieved. When the coupling strength K is very small, the time evolution of each oscillator is dominated by its own natural intrinsic frequency; thus, the oscillators are asynchronous. As K is increased sufficiently further, one expects a synchronous state of phase oscillators to develop with $0 < \bar{\Delta} < 1$. It is noteworthy that the perfectly synchronized state with $\bar{\Delta} = 1$ cannot occur at any finite coupling strength for nonidentical oscillators described by Eq. (1), in sharp contrast to a system of identical oscillators.

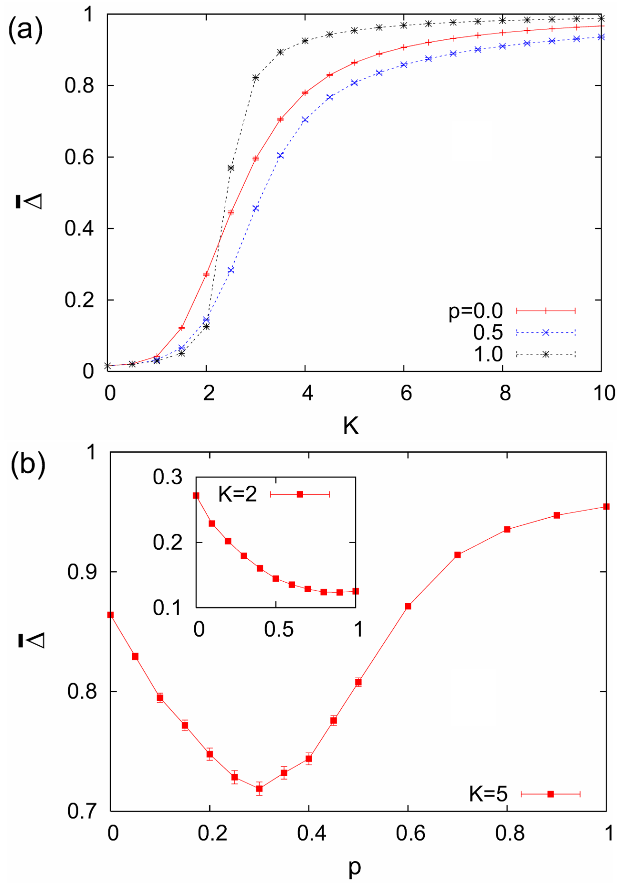

Fig. 2. (Color online) (a) $\bar{\Delta}$ versus K at $p = 0.0, 0.5$ and 1.0 for the network with $L = 8$ and $b = 3$ ($N = 3280$). (b) $\bar{\Delta}$ versus p at $K = 5$. $\bar{\Delta}$ becomes smaller and then increases as p is increased. Inset: $\bar{\Delta}$ versus p at $K = 2$. In contrast to $K = 5$, $\bar{\Delta}$ is observed to decrease as p is increased.

III. RESULT

We construct networks for $L = 6$, 8 and 10, corresponding to $N = 364$, 3280 and 29524, respectively, at various values of p with the branching ratio $b = 3$. We then integrate the equations of motion in Eq. (1) via the second-order Runge-Kutta algorithm with the discrete time step $dt = 0.05$ and $\Delta(t)$ in Eq. (2), averaged over $10^2 \sim 10^3$ different realizations of network structures and intrinsic frequencies, is measured as a function of time t. The time average $\bar{\Delta}$ is measured after a sufficiently long time 500.

In Figure 2(a), we plot $\bar{\Delta}$ versus K for $L = 8$ [$N = (3^8 - 1)/2 = 3280$] at various values of p. For all curves, as K is increased, $\bar{\Delta}$ is also increased, as expected. The curve for $p = 0.5$ is found to be below the curve at $p = 0$ at any K, which indicates that the split of directed edges with a fixed incoming strength makes the network less synchronizable. In contrast, at a sufficiently large K, as p is increased further, $\bar{\Delta}$ begins to increase with p. Surprisingly, at $p = 1.0$, the network exhibits better synchronization then the network at $p = 0$. This interesting nonmonotonic behavior of $\bar{\Delta}(p)$ is more clearly seen in Figure 2(b) for $K = 5.0$, which exhibits a minimum at $p \approx 0.3$; then, $\bar{\Delta}(p)$ increases beyond $\bar{\Delta}(p = 0)$. At small K, *e.g.*, $K = 2$, $\bar{\Delta}$ decreases when p is increased, as shown in the inset of Figure 2(b).

We believe that our results obtained above for the synchronization of nonidentical oscillators are very interesting and important in a general context. As more edges are split in a hierarchical tree network, the global occurrence of synchronization first becomes worse due to the inefficiency of bidirectional information flow; however, as more and more split edges are made, the synchronization behavior becomes better than that of a unidirectional hierarchical tree structure. This may draw some attention not only from the physics community but also from other disciplines, like political science and organization theory.

To understand the nonmonotonic behavior of the synchronization order parameter, we first analytically investigate the tree network at $p = 0$ within the linear approximation. Let us start from a directed linear chain of N oscillators ($b = 1$), described by [see Eq. (1)]

$$\dot{\theta}_i = \Omega_i + K \sin(\theta_{i-1} - \theta_i), \tag{3}$$

for $i > 1$ and $\dot{\theta}_1 = \Omega_1$. For convenience, we set $\Omega_1 = 0$ and $\theta_1 = 0$. When $K \gg 1$, it is straightforward to get the steady-state solution $\theta_i = \sum_{j=2}^{i} \sin^{-1}(\Omega_j/K) \approx (1/K)\sum_{j=2}^{i}\Omega_j$, which leads to

$$\Delta_{\rm chain} \approx \frac{1}{N}\frac{1 - e^{-N/2K^2}}{1 - e^{-1/2K^2}} \tag{4}$$

for a linear chain of size N. Here, we define the complex order parameter $\Delta_{\rm chain} = (1/N)\left\langle \sum_i e^{i\theta_i}\right\rangle$ and perform a Gaussian integral over every Ω_i. A similar analytic calculation applied to the tree network at $p = 0$ with a branching ratio b gives [see Figure 1(a)]

$$\Delta_{\rm tree} \approx \frac{1}{N}\frac{(be^{-1/2K^2})^L - 1}{be^{-1/2K^2} - 1}. \tag{5}$$

In Eqs. (4) and (5), the order parameters $\Delta_{\rm chain}$ and $\Delta_{\rm tree}$ vanish when $N \to \infty$ at any finite K. Consistent with $\Delta_{\rm tree}$, $\bar{\Delta}$ at $p = 0$ decreases when L is increased, as displayed in Figure 3 for $b = 3$. Consequently, we conclude that there is no synchronous state for the pure tree network ($p = 0$) in the thermodynamic limit, as analytically shown in Eq. (5), which is consistent with numerical result at $p = 0$ in Figure 3. Whereas $\Delta_{\rm chain} \sim 1/N$ for the linear chain, as shown in Eq. (4), the pure tree network with $b > 1$ exhibits $\Delta_{\rm tree} \sim N^{-1/2K^2 \ln b}$ when the inequality $be^{-1/2K^2} > 1$ is satisfied [see Eq. (5)]. It is noteworthy that for a sufficiently large value of K, $\Delta_{\rm tree}$ in Eq. (5) decays very slowly and even the system composed of Avogadro's number of oscillators can have a nonvanishing value of $\bar{\Delta}$ far from zero in the directed tree network.

We below try to generalize the above argument for the pure tree to include the effect of split edges. In the following discussion for the synchronization behavior at small p, we conceptually divide vertices in the network into two groups: The first group contains vertices oriented from the top vertex whose incoming edges are not split and, thus, have unit weights and the other group contains all other vertices, which do not belong to the first group. Since all edges in the first group have twice bigger weights than those in the second group, we assume that the first group works as a synchronized backbone tree while the vertices in the second group are less synchronized. The number of vertices in the first group of the synchronized backbone tree is written as $N_{\rm tree}$: When $p = 0$, all vertices in the network are connected by nonsplit edges with unit weights; thus, $N_{\rm tree} = N$. In the opposite limit of $p \to 1$, we expect $N_{\rm tree} = O(1)$ since most edges are split.

With the simple, but plausible, assumption that those $N_{\rm tree}$ oscillators are more strongly coupled to each other and, thus, provide a major contribution to the synchronization order parameter, we obtain $\langle e^{i\theta_i}\rangle \approx (1-p)^l e^{-l/2K^2}$ because the probability that a vertex in the level l ($l = 0, 1 \ldots L-1$) belongs to the backbone tree is $(1-p)^l$. The estimate of the synchronization order parameter then follows as

$$\Delta'_{\rm tree} \approx \frac{1}{N}\frac{[b(1-p)e^{-1/2K^2}]^L - 1}{b(1-p)e^{-1/2K^2} - 1}. \quad (6)$$

The above calculation obviously underestimates the actual value of the synchronization order parameter because it assumes that vertices outside of the synchronized backbone tree do not contribute to the order parameter at all. When L is sufficiently large, $\Delta'_{\rm tree} \sim \Delta_{\rm tree}(1-p)^L$ at small p because $N \sim b^L$ and $\Delta_{\rm tree} \sim e^{-L/2K^2}$. We display $\bar{\Delta}/\bar{\Delta}(0)$ as a function of p for $K = 2$ in the inset of Figure 3(a); $\bar{\Delta}/\bar{\Delta}(0)$ is observed to decrease when p is increased and to vanish when L is increased, which is consistent with $\Delta'_{\rm tree}/\Delta_{\rm tree} \sim (1-p)^L$ at small p.

The monotonic decrease of $\bar{\Delta}$ for small K is also explained as follows: Even when a network of the infinite size is fully asynchronous ($\Delta = 0$) at given K, corresponding networks of finite sizes may exhibit partial synchronization due to finite-size effects. In this case, we expect that if p is increased and, thus, more edges are split, the synchronization will be worsened because these added edges of half weights will reduce the number of vertices in the synchronized backbone tree. This indicates that as p is increased, the synchronization order parameter is expected to decrease, as is shown in Figure 3(a).

Even at large K $\bar{\Delta}/\bar{\Delta}(0)$ is found to decrease as L is increased when p is small for $K = 4$ and 6, as seen in the insets of Figures 3(b) and (c). Note that the worse synchronization at $p \neq 0$ occurs in finite systems because $\Delta'_{\rm tree} \to 0$ as $L \to \infty$, which is consistent with the results in Figure 3; when L is increased, $\bar{\Delta}$ vanishes at all p for $K = 2$ and the minimum point of $\bar{\Delta}$ decreases and shifts toward $p = 0$ for $K = 4$ and 6.

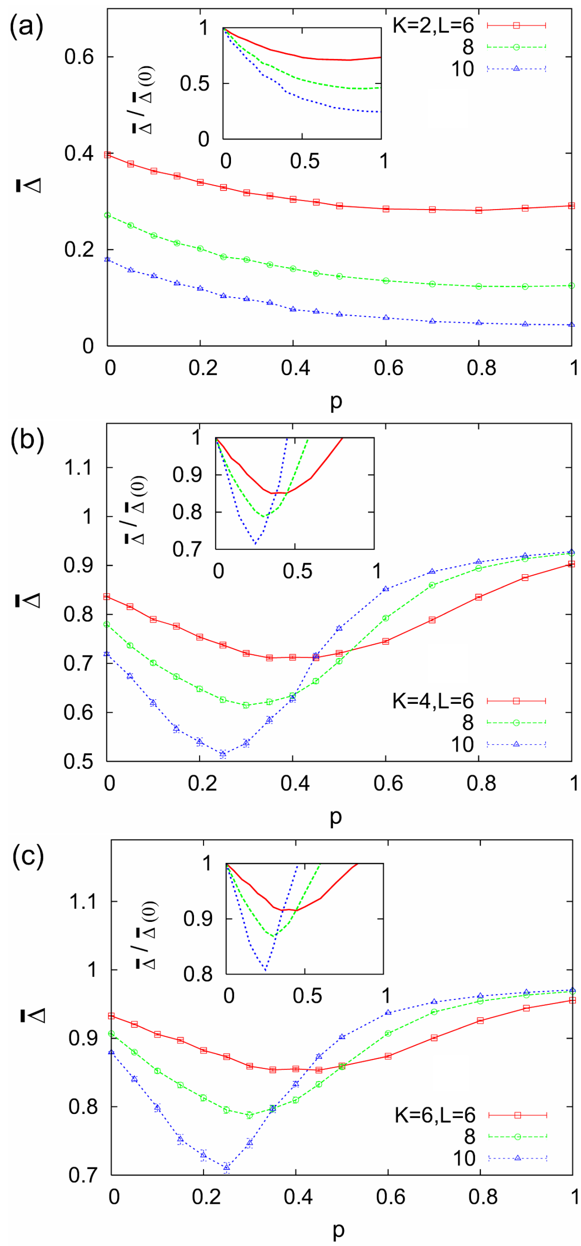

Fig. 3. (Color online) (a) $\bar{\Delta}$ as a function of p for $L = 6, 8$ and 10 with $b = 3$ at $K = 2.0$. $\bar{\Delta}$ is observed to decrease as p is increased and vanishes as L is increased. Corresponding plots for (b) $K = 4.0$ and (c) $K = 6.0$. All curves show nonmonotonic behaviors with respect to p. However, the minimum value of $\bar{\Delta}$ decreases and the minimum position shifts toward $p = 0$ as L is increased. Near $p = 1$, in contrast, $\bar{\Delta}$ increases as L is increased. The insets in (a), (b) and (c) show that normalized order parameter $\bar{\Delta}(p)/\bar{\Delta}(p = 0)$.

We next investigate numerically the synchronization phase transition near $p = 1$ by employing the finite-size

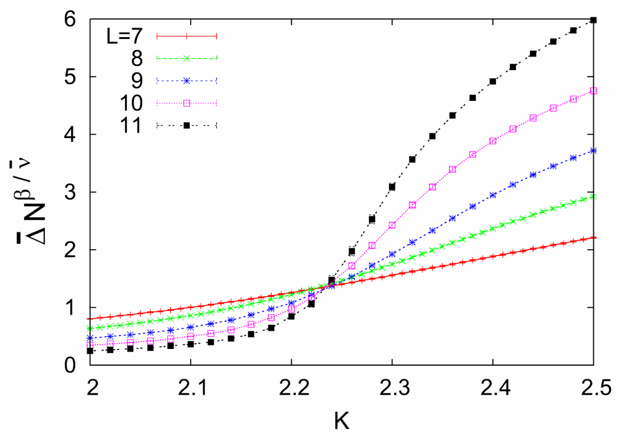

Fig. 4. (Color online) Scaling plot for $\bar{\Delta}N^{\beta/\bar{\nu}}$ vs K at $p = 1.0$ for $b = 3$. When $\beta/\bar{\nu} = 0.2$, which is the mean field value, crossing occurs at $K \approx 2.23$.

scaling form [12,13]

$$\bar{\Delta} = N^{-\beta/\bar{\nu}} f\left[(K - K_c)N^{1/\bar{\nu}}\right], \quad (7)$$

where β and $\bar{\nu}$ are critical exponents and K_c is the critical coupling strength. The critical exponents $\bar{\nu} = 5/2$ and $\beta = 1/2$ have been recently proposed for globally-coupled Kuramoto-type oscillators [12]. We plot in Figure 4 $\bar{\Delta}N^{\beta/\bar{\nu}}$ versus K at $p = 1$ for the directed network. With $\beta/\bar{\nu} = (1/2)/(5/2) = 1/5$, the well-defined unique crossing of curves occurs at $K \approx 2.23$ (Figure 4). We, thus, conclude that our directed network at sufficiently large value of splitting probability p has a synchronization transition, which belongs to the same universality class as that for the globally-coupled oscillators [12]. The existence of the phase transition at a finite coupling strength at $p = 1$ and the absence at $p = 0$ provide us an explanation of $\bar{\Delta}(p = 0) < \bar{\Delta}(p = 1)$: The absence of the phase transition at $p = 0$ implies that $\bar{\Delta}(p = 0)$ is reduced toward zero as the network size is increased whereas the existence of phase transition for $p = 1$ indicates $\bar{\Delta}(p = 1)$ is finite for $K > K_c$. Consequently, for sufficiently large networks, the inequality $\bar{\Delta}(p = 0) < \bar{\Delta}(p = 1)$ should be fulfilled for any coupling strength larger than K_c. The result presented in Figure 2(a) coincides very well with the above expectation: As K is increased beyond some value, $\bar{\Delta}(p = 1)$ becomes larger than $\bar{\Delta}(p = 0)$.

Applying the finite-size scaling method for the directed networks at other values of p, we find that the mean-field exponents $\beta = 1/2$ and $\bar{\nu} = 5/2$ persist over a broad range of p. The critical coupling strength $K_c(p)$ is observed to increase as p is lowered, *e.g.*, $K_c \approx 2.31$ and 2.36 at $p = 0.9$ and 0.8, respectively, consistent with the fact that $\bar{\Delta}$ decreases at all p as L is increased for $K = 2$, as shown in Figure 3(a). The decrease of $K_c(p)$ makes $K - K_c$ bigger for a given K, giving us the reason $\bar{\Delta}$ increases with p near $p = 1$, as shown in Figures 2 and 3. Due to the severe finite-size effect, the vanishing of $\bar{\Delta}$ in a disordered phase can be seen only in extremely huge networks [see Eq. (5) and related discussion above], so we were not able to study the network at small p. Although we have not studied our network model at even smaller values of p because of the practical difficulty of using bigger sizes, the synchronization behavior in the present work is very similar to that reported in a previous study for undirected small-world networks [11], in which $K_c(p \to 0) \to \infty$ was concluded and $K_c(p)$ was shown to decrease as p is increased.

IV. SUMMARY

We numerically studied the synchronization of Kuramoto-type nonidentical oscillators on directed networks which embed the directed tree. Independently of the optimal conditions made for identical oscillators in the directed networks embedding the hierarchical structure [5], we found in this work that the synchronization of Kuramoto-type phase oscillators is significantly enhanced as sufficiently many edges are split with the input strength of each vertex remaining unchanged. Although systems with large numbers of oscillators have a nonvanishing value of the order parameter in the pure tree, there is no synchronous state in the thermodynamic limit. In contrast to the case of the pure tree, in directed networks constructed via edge splittings, partial synchronization begins to develop at a finite coupling constant, yielding better synchronization than directed tree networks, even for finite systems. We also have revealed that the critical coupling constant K_c decreases as the splitting probability p is increased, although K_c at $p \approx 0$ needs further study. The implication one can draw from this work is that a collective emergent behavior can happen on a much bigger scale if sufficiently many bidirectional channels of information transfer exist.

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