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Phase Transitions in Positionally Disordered Josephson Junction Arrays in a Magnetic Field

Jaegon Um
National Creative Research Initiative Center for Superconductivity, POSTECH, Pohang 790-784

Sung-Ik Lee
National Creative Research Initiative Center for Superconductivity, POSTECH, Pohang 790-784 and Quantum Material Laboratory, Korea Basic Science Institute, Daejeon 305-333

Beom Jun Kim*
Department of Physics, Sungkyunkwan University, Suwon 440-746

We numerically investigate phase transitions of two-dimensional (2D) Josephson junction arrays (JJA) with positional disorder in a transverse magnetic field. The current-voltage characteristics in strong disorder are computed. It is revealed that there is a phase transition of a non-Kosterlitz-Thouless type at a finite temperature, consistent with a recent experiment on the positionally disordered JJA as well as previous numerical studies of a 2D gauge glass model.

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I. INTRODUCTION

A two-dimensional (2D) random gauge XY model has been studied as a theoretical realization of a 2D Josephson junction arrays (JJA) with positional disorder in the presence of an external transverse magnetic field [1, 2]. When the frustration f, which is the number of magnetic flux quanta per plaquette of JJA, takes an integer value, the magnetic bond angle A_{ij} becomes a quenched random variable due to the random positions of sites i and j, and the Hamiltonian is given by [1–3]

$$H = -J \sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j - A_{ij}\right), \tag{1}$$

where J is the Josephson coupling strength, θ_i is the phase angle at i, and $A_{ij} \in [-r\pi, r\pi]$ with the disorder strength r ($0 \leq r \leq 1$). In the case of the positionally disordered JJA (PDJJA), $r \approx f\Delta$ [3–6], where Δ is the positional disorder strength described below.

In the weak disorder ($r < r_c$ with the critical disorder strength r_c), it has been unanimously revealed that there exists a Kosterlitz-Thouless (KT)-type phase transition at the critical temperature T_c, which decreases as r becomes larger [2,7,8]. In contrast, in the strong disorder regime ($r > r_c$), the positional disorder can make the vortex motion [9] sluggish, which opens a possibility of the low-temperature superconducting glass phase. The existence of the superconducting order at nonzero temperatures is still under strong controversy. Especially, the 2D gauge glass model corresponding to the strong disorder limit ($r = 1$) has drawn much interest [2, 7, 8]. Recently, numerical studies of the 2D gauge glass model have shown that there exists a non-KT-type phase transition at a finite temperature, through the computations of the glass susceptibility [10], the correlation function [10], the linear resistance [11], and the current-voltage (IV) characteristics [13], leading to the estimations $T_c = 0.22$ (unit of J/k_B with the Boltzmann constant k_B), the dynamic critical exponent $z = 2.0$, and the correlation-length critical exponent $\nu = 1.2$. Very recently, the 2D PDJJA has been studied in experiments and the phase diagram in the plane of the disorder strength and the temperature has been constructed, which provides strong evidence of the existence of a superconducting phase at nonzero temperatures in the strong-disorder regime [12].

In this paper, we perform the resistively shunted junction (RSJ) simulation of the positionally disordered JJA (PDJJA) in an extensive scale to calculate IV curves at strong disorder ($r > r_c$), in order to compare these with the existing studies of the 2D gauge glass model as well as the static Monte-Carlo simulation studies of the PDJJA [3, 6]. The paper is organized as follows: The PDJJA model and the numerical methods employed are briefly explained in Section II. Section III is devoted to the main results of the present study, the IV characteristics, followed by concluding remarks in Section IV.

*E-mail: beomjun@skku.edu

II. MODEL AND NUMERICAL CALCULATION

On the assumption that the presence of positional disorder does not affect the coupling energy J, the Hamiltonian of 2D PDJJA in an external magnetic field $\mathbf{B} = B\hat{z}$ takes the same form as in Eq. (1). The magnetic bond angle A_{ij} is obtained from the line integral of the magnetic vector potential [3,6]:

$$A_{ij} = \frac{Ba^2\pi}{\Phi_0}(x_j + x_i)(y_j + y_i), \quad (2)$$

where a is the lattice constant (set to unity henceforth) and Φ_0 is the flux quantum. The position of the site i is written as

$$\mathbf{r}_i \equiv (x_i, y_i) = \left(x_i^0 + \delta x_i, y_i^0 + \delta y_i\right), \quad (3)$$

where $\mathbf{r}_i^0 \equiv \left(x_i^0, y_i^0\right)$ is the original position of the lattice site without positional disorder, and δx_i (δy_i) is the random quenched variable in $[-\Delta, \Delta]$ with uniform probability distribution. The disorder average of the sum of the magnetic bond angles around one plaquette is given by

$$\left[\sum_p A_{ij}\right] = 2\frac{B}{\Phi_0}\pi = 2\pi f, \quad (4)$$

where we define f as the average frustration.

On introducing the twist variable, $\mathbf{D} = (D_x, D_y)$ for the fluctuating twist boundary condition, the Hamiltonian takes the form [14]

$$H = -J\sum_{\langle ij\rangle}\cos(\theta_i - \theta_j - D_{ij} - A_{ij}), \quad (5)$$

where $D_{ij} = \mathbf{D}\cdot\left(\mathbf{r}_j^0 - \mathbf{r}_i^0\right)$. To calculate the voltage, we use RSJ dynamics [14] in an $L \times L$ array of resistively shunted junctions. From local current conservation, the equation of motion for the phase angle θ_i is given by

$$\dot{\theta}_i = -\sum_j G_{ij}\sum_k{}' \left[\sin(\theta_j - \theta_k - D_{jk} - A_{jk}) + \eta_{jk}\right], \quad (6)$$

where $\sum_k'$ denotes the summation of the nearest neighbor sites of j, G_{ij} is the lattice Green's function, and η_{jk} the ensemble averages $\langle\cdots\rangle$ of the thermal current satisfying

$$\begin{aligned} \langle\eta_{ij}(t)\rangle &= 0, \\ \langle\eta_{ij}(t)\eta_{kl}(0)\rangle &= 2T(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\,\delta(t) \end{aligned} \quad (7)$$

at temperature T, in units of J/k_B. The equation of motion for the twist variable can be obtained by the global current conservation: when external dc current i_d exists in x-direction,

$$\begin{aligned} \dot{D}_x &= \frac{1}{L^2}\sum_{\langle i,j\rangle x}\left[\sin(\theta_i - \theta_j - D_x - A_{ij})\right] + \eta_{D_x} - i_d, \\ \dot{D}_y &= \frac{1}{L^2}\sum_{\langle i,j\rangle y}\left[\sin(\theta_i - \theta_j - D_y - A_{ij})\right] + \eta_{D_y}, \end{aligned} \quad (8)$$

where $\sum_{\langle i,j\rangle x(y)}$ denotes the summation of the nearest neighbor sites along $x(y)$-direction and the thermal noise $\eta_{D_{x(y)}}$ satisfies $\langle\eta_{D_{x(y)}}(t)\rangle = 0$ and

$$\langle\eta_{D_x}(t)\eta_{D_x}(0)\rangle = \langle\eta_{D_y}(t)\eta_{D_y}(0)\rangle = 2\frac{T}{L^2}\delta(t). \quad (9)$$

The voltage is then obtained from the Josephson voltage relation $V_x = -\frac{\hbar L}{2e}\dot{D}_x$.

For an efficient numerical integration of equations of motion through the use of the fast Fourier transformation, the periodic boundary condition is required for Eq. (6). However, if we simply employ the usual PBC $\theta_i = \theta_{i+L\hat{x}} = \theta_{i+L\hat{y}}$, the right-hand side of Eq. (6) violates PBC since $A_{i+L\hat{x}\ j+L\hat{x}} \neq A_{ij}$. In order to remedy this, we modify the boundary condition to

$$\begin{aligned} \theta_{i+L\hat{x}} &= \theta_i - 2\pi f L\delta y_i, \\ \theta_{i+L\hat{y}} &= \theta_i, \end{aligned} \quad (10)$$

which makes the dynamics of phase angles and the twist variables unchanged since $2\pi f L\delta y_i$ is constant in time.

We perform the simulations in 128×128 arrays for only one disorder realization expecting the self-averaging effect, which can be justified in comparison with existing experiments on the 200×800 PDJJA in Ref. 12.

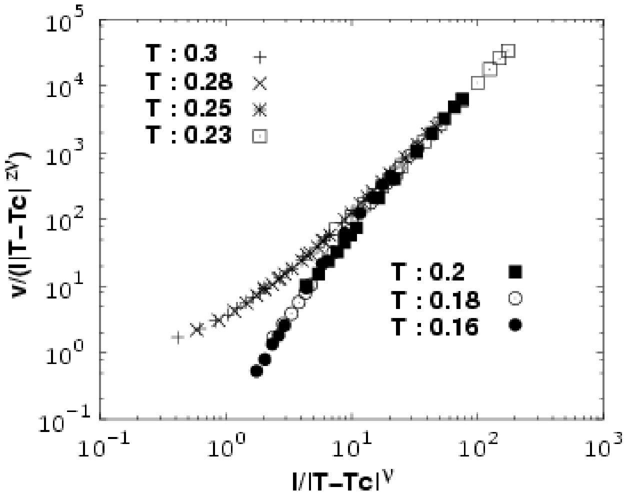

Fig. 1. Scaled IV curves for the positionally disordered JJA at $\Delta = 0.2$ and $f = 4$, corresponding to the strongly disordered case. The use of $T_c = 0.22$, $z = 2$, and $\nu = 1.2$ makes the IV data collapse to two well-separated curves, strongly indicating the existence of a superconducting-normal transition at finite T_c of the non-KT type.

III. RESULTS

In our RSJ simulations, we set the positional disorder strength $\Delta = 0.2$ and the frustration $f = 4$, corresponding to the effective disorder strength [3–6] $f\Delta = 0.8$, which clearly belongs to the strong disorder regime $r > r_c$ in the 2D random gauge XY model, since $r_c \approx 0.4$ [2,7,8]. We calculate the IV curves at various temperatures, from $T = 0.3$ to 0.16, and then use the scaling form [15]

$$V = I\,|T - T_c|^{z\nu}\,F_{\pm}\left(I\,|T - T_c|^{-\nu}\right), \tag{11}$$

where $F_{\pm}(x)$ is the scaling function in the high(low)-temperature phase. In Figure 1, it is clearly observed that numerical data obtained at various temperatures and external currents collapse to two well-separated curves, which is very strong evidence supporting the existence of a phase transition at T_c of the non-KT type. From the scaling collapse, we estimate the critical temperature $T_c = 0.22$, together with the dynamic critical exponent $z = 2.0$ and the critical exponent $\nu = 1.2$. As expected, these values are consistent with previous numerical studies of the 2D gauge glass model [10,11,13], signalling that the PDJJA and the gauge glass model belong to the same universality class in the strong-disorder regime.

IV. CONCLUSIONS

From the IV characteristics of the positionally disordered JJA calculated by RSJ dynamics, we have found that a finite-temperature phase transition of non-KT type exists at strong disorder. This is consistent with numerical work [8,10,11,13] which predicted that there exists a finite-temperature superconducting order in the 2D random gauge XY model at $r > r_c$ and the 2D gauge glass model. Furthermore, we have shown that the critical temperature and the critical exponents are the same as the values predicted in the numerical work on the 2D gauge glass model [10,11,13]. The result of the existence of superconducting phase at strong disorder is also in good agreement with a recent experiment on the PDJJA [12]. Although the critical temperature and the dynamic critical exponent $z = 1.8(3)$ in Ref. 12 are consistent with our findings, the difference between $\nu = 2.0(3)$ in Ref. 12 and $\nu \approx 1.2$ in this work still remains to be understood.

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