Quantum fluctuations in superconducting arrays with a general capacitance matrix

Beom Jun Kim and M. Y. Choi
Department of Physics, Seoul National University, Seoul 151-742, Korea
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We study dissipation and quantum-fluctuation effects on the critical behavior of a superconducting array which has both self-capacitance and junction capacitance. The effective classical Hamiltonian is obtained through the use of the variational method in the path-integral formalism and then the renormalization-group technique is applied. A reentrant transition is found to exist for moderate quantum fluctuations; it persists regardless of the ratio of the junction capacitance to the self-capacitance although the junction capacitance tends to suppress the reduction of the transition temperature due to quantum fluctuations. As the strength of dissipation is increased, the reentrant behavior becomes weaker. The phase boundaries at zero temperature are obtained and compared with the results of the other mean-field approach. The system in the presence of an external magnetic field is also studied.

There has been considerable interest in charging energy effects on the critical behavior of two-dimensional (2D) superconducting arrays, weakly coupled by Josephson junctions.[1] Since the relevant variables in such arrays are collective degrees of freedom, the phases of the superconducting order parameters, they provide model systems for understanding quantum effects on a macroscopic scale. Here quantum fluctuations in general tend to suppress superconductivity, leading to the possibility of reentrance into the normal state at low temperatures; the existence of such reentrance, particularly *in the self-charging limit*, has been a source of controversy.[2–4] Recently, it has become possible to fabricate superconducting arrays with small capacitances which lead to strong charging effects.[5] In these arrays the self-capacitance C_0 is no longer dominant over the junction capacitance C_1, thus making it necessary to consider the effects of both capacitances together. Arrays with discrete charge and a general capacitance matrix, which contains both C_0 and C_1, have been studied via the coarse-graining approach, which is mean-field-like in nature.[6] However, the mean-field approximation in general does not take into account thermal fluctuations correctly, and is not expected to give reliable results in two dimensions, where fluctuations are strong. In particular, it is obvious that the mean-field approximation cannot describe the subtle character of the Kosterlitz-Thouless (KT) transition[1] present in the system without charging effects. For example, when $C_0 = 0$, the coarse-graining approach leads to the unphysical result that the system remains in the disordered phase for arbitrarily large values of the Josephson coupling.[1] Further, if the leakage currents are not strictly zero, which is the usual experimental situation, the Ohmic dissipation allows continuous flow of charge.[7,8] Thus in this case we need to consider arrays with continuous charge states and to examine the effects of dissipation together with quantum fluctuations beyond mean-field theory. Such dissipation effects have been studied, again in the self-charging limit, and found to suppress quantum fluctuations.[9–11]

This paper investigates quantum-fluctuation and dissipation effects on the critical behavior of 2D superconducting arrays, with emphasis on the roles of the self-capacitances and junction capacitances. We consider arrays with a general capacitance matrix and allow continuous charge states due to the Ohmic dissipation. To study the system beyond mean-field theory, we apply the renormalization-group (RG) technique to the effective Hamiltonian obtained by the path-integral formalism;[12] the path integral is computed via the Giachetti-Tognetti-Feynman-Kleinert (GTFK) variational method.[13] It should be noted that although this method is semiclassical in nature, it sometimes gives very reliable results even at zero temperature. In particular, this approach applied to the self-charging limit indeed produced results consistent with the known zero-temperature result.[4] This is in contrast with other semiclassical calculations adopting conventional perturbational or variational methods, which are in general inaccurate at low temperatures and lead to incorrect results at zero temperature.[3] Here the method of GTFK again reveals a reentrant transition for an intermediate amount of quantum fluctuations. While the self-capacitance gives rise to quantum fluctuations, the junction capacitance is found to suppress some effects of them. Nevertheless, the reentrance persists regardless of the ratio of the junction capacitance to the self-capacitance. Dissipation also tends to suppress quantum fluctuations: As the strength of dissipation is increased, the reentrant behavior becomes weaker. To consider the influence of an external magnetic field, we also investigate the fully frustrated system and find that these behaviors due to quantum fluctuations and dissipation in general remain qualitatively the same, displaying a similar structure of the phase diagram.

We begin with the Hamiltonian describing a 2D square array of Josephson junctions:

$$H = \frac{1}{2}\sum_{i,j}(q_i - q_0)C_{ij}^{-1}(q_j - q_0) - J\sum_{\langle i,j\rangle}\cos(\phi_i - \phi_j), \tag{1}$$

where q_i and ϕ_i represent the charge and phase of the Cooper pairs at site i, respectively, satisfying the commutation relation $[\phi_i, q_i] = 2ei$, J denotes the Josephson coupling strength, and the capacitance matrix is given by

$$C_{ij} = (C_0+4C_1)\delta_{i,j} - C_1(\delta_{i,j+\mathrm{x}}+\delta_{i,j-\mathrm{x}}+\delta_{i,j+\mathrm{y}}+\delta_{i,j-\mathrm{y}}),$$

with self-capacitance C_0 and junction capacitance C_1. The charge frustration q_0 may be introduced to the system by applying the potential difference V_0 between the array and the substrate: $q_0 = C_0V_0$.[6]

A convenient way to deal with such a quantum mechanical Hamiltonian is the Feynman path-integral formalism.[12] In the formalism, the partition function of the system is expressed in terms of the path integral

$$Z = \prod_i \int d\phi_i(0) \int \mathcal{D}\phi_i(\tau)\exp\{-S[\phi_i(\tau)]\}, \tag{2}$$

with the action

$$S = \int_0^\beta d\tau\left\{\sum_{i,j}\frac{1}{2}\dot{\phi}_i(\tau)M_{ij}\dot{\phi}_j(\tau) - J\sum_{\langle i,j\rangle}\cos[\phi_i(\tau) - \phi_j(\tau)]\right\} + S_D, \tag{3}$$

where we have set $\hbar \equiv 1$, $\beta \equiv 1/k_BT$, and $M_{ij} \equiv C_{ij}/4e^2$, and the path integral includes the sum over periodic paths satisfying $\phi_i(\beta) = \phi_i(0)$. S_D is the effective dissipative action, which, for the Ohmic dissipation via the shunt resistance R_s, may be taken as[8,14]

$$S_D = \frac{\gamma}{8\beta^2}\sum_{\langle i,j\rangle}\int_0^\beta d\tau\int_0^\beta d\tau' \times\frac{[\phi_i(\tau) - \phi_j(\tau) - \phi_i(\tau') + \phi_j(\tau')]^2}{\sin^2[\pi(\tau - \tau')/\beta]}, \tag{4}$$

with $\gamma \equiv h/4e^2R_s$ measuring the strength of dissipation.

Note that the charge frustration q_0 does not appear in the action S given by Eq. (3), which results from the periodicity $\int_0^\beta d\tau\dot{\phi}_i(\tau) = 0$. Consequently, the charge frustration here plays no role in the phase transition of the system. In the absence of continuous charge flow, on the other hand, only discrete charge states are allowed and all the paths satisfying $\phi_i(\beta) = \phi_i(0) + 2\pi n_i$ with integer n_i contribute to the path integral in Eq. (2).[8] In this case the path integral should include the summation over the winding number n_i, yielding interesting behaviors associated with the charge frustration.[6] Here, however, the Ohmic dissipation due to leakage currents allows us to regard the charge as a continuous variable, and only those paths with winding number $n_i = 0$ contribute to the path integral.

Following the GTFK method we choose a trial action of the form[11,13]

$$S_1 = \int_0^\beta\left\{\frac{1}{2}\sum_{i,j}\dot{\phi}_i(\tau)M_{ij}\dot{\phi}_j(\tau) + \frac{1}{2}\sum_{i,j}[\phi_i(\tau) - \phi_{i,0}]D_{ij}(\vec{\phi}_0)[\phi_j(\tau) - \phi_{j,0}] + L(\vec{\phi}_0)\right\}d\tau + S_D, \tag{5}$$

where $\vec{\phi}_0 \equiv \{\phi_{i,0}\}$ denotes the zero-frequency components in the Fourier decomposition of $\phi_i(\tau)$:

$$\phi_i(\tau) = \phi_{i,0} + \sum_{n=1}^{\infty}\left[\phi_{i,n}\exp(i\omega_n\tau) + \phi_{i,n}^*\exp(-i\omega_n\tau)\right], \tag{6}$$

with the Matsubara frequencies $\omega_n \equiv 2\pi n/\beta$, and $D_{ij}(\vec{\phi}_0)$ and $L(\vec{\phi}_0)$ are functions to be determined via the extremal principle.[12] Noting the translational symmetry of the action in Eq. (3), we may choose the translationally invariant form for D_{ij}:

$$D_{ij}(\vec{\phi}_0) = D_0(\vec{\phi}_0)\delta_{ij} + D_1(\vec{\phi}_0)\delta_{ij'},$$

where j' represents nearest neighbors of site j. In terms of the Fourier components given by Eq. (6), the trial action (4) is expressed as

$$S_1 = \beta\sum_{i,j}\sum_{n=1}^{\infty}\left[\omega_n^2M_{ij} + \omega_n\Gamma_{ij} + D_{ij}\right]\phi_{i,n}\phi_{j,n}^* + \beta L(\vec{\phi}_0), \tag{7}$$

with

$$\Gamma_{ij} \equiv \frac{\gamma}{2\pi}(4\delta_{ij} - \delta_{ij'}).$$

The corresponding trial partition function thus becomes

$$Z_1 = \prod_i\int\mathcal{D}\phi_i(\tau)\exp(-S_1) = \int\prod_i\frac{d\phi_{i,0}}{\sqrt{2\pi\beta m_i^{-1}}}\exp\left[-\sum_{n=1}^{\infty}\ln\det B_n - \beta L(\vec{\phi}_0)\right] \equiv \int\prod_i\frac{d\phi_{i,0}}{\sqrt{2\pi\beta m_i^{-1}}}\exp(-\beta H_1), \tag{8}$$

where $B_n \equiv \omega_n^{-2}(\omega_n^2 M + \omega_n \Gamma + D)M^{-1} \equiv \omega_n^{-2}\tilde{B}_n M^{-1}$ in matrix notation and m_i is the ith eigenvalue of M. The average with respect to the trial Hamiltonian H_1 of the difference between the true action S and the trial action S_1 then takes the form

$$\frac{1}{\beta}\langle S - S_1\rangle_1 \equiv \frac{1}{Z_1}\int\prod_i \frac{d\phi_{i,0}}{\sqrt{2\pi\beta m_i^{-1}}}\exp(-\beta H_1)\frac{1}{\beta}(S-S_1)$$
$$= \frac{1}{Z_1}\int\prod_i \frac{d\phi_{i,0}}{\sqrt{2\pi\beta m_i^{-1}}}\exp\left[-\sum_{n=1}^{\infty}\ln\det B_n - \beta L(\vec{\phi}_0)\right]\left[V_{\rm sm}(\vec{\phi}_0) - \frac{1}{\beta}\sum_n \mathrm{Tr}\, D\tilde{B}_n^{-1} - L(\vec{\phi}_0)\right],$$

where $\mathrm{Tr}\, D\tilde{B}_n^{-1}$ indicates the trace of the matrix $D\tilde{B}_n^{-1}$ and $V_{\rm sm}(\vec{\phi}_0)$ is the smeared potential given by

$$V_{\rm sm} \equiv -J\sum_{\langle i,j\rangle}\cos(\phi_{i,0}-\phi_{j,0})\exp\left\{-\frac{1}{\beta}\sum_n\left[(\tilde{B}_n^{-1})_{ii} + (\tilde{B}_n^{-1})_{jj} - (\tilde{B}_n^{-1})_{ij} - (\tilde{B}_n^{-1})_{ji}\right]\right\}.$$

In the GTFK formalism the approximate free energy obtains from the minimization of $-\ln Z_1 + \langle S - S_1\rangle_1$; it yields an upper bound of the true free energy. First of all, minimization with respect to the variation of $L(\vec{\phi}_0)$ leads to the relation

$$L(\vec{\phi}_0) = V_{\rm sm}(\vec{\phi}_0) - \frac{1}{\beta}\sum_n \mathrm{Tr}\, D\tilde{B}_n^{-1},$$

which in turn gives the trial Hamiltonian H_1 in the form

$$\beta H_1 = \sum_{n=1}^{\infty}\ln\det B_n - \beta\mathrm{Tr}\, DG - \beta J\sum_{\langle i,j\rangle}\cos(\phi_{i,0}-\phi_{j,0})\exp\left[-(G_{ii}+G_{jj}-G_{ij}-G_{ji})\right], \tag{9}$$

where the matrix G is defined by

$$G \equiv \frac{1}{\beta}\sum_{n=1}^{\infty}\tilde{B}_n^{-1} = \frac{1}{\beta}M^{-1}\sum_n \frac{1}{\omega_n^2}B_n^{-1}. \tag{10}$$

The desired effective classical partition function is given by

$$Z_{\rm cl} = \int\prod_i \frac{d\phi_i}{\sqrt{2\pi\beta m_i^{-1}}}\exp(-\beta H_{\rm cl}),$$

where the effective classical Hamiltonian $H_{\rm cl}$ is obtained by minimizing H_1 with respect to D_{ij}:

$$H_{\rm cl} = \frac{1}{\beta}\sum_{n=1}^{\infty}\ln\det B_n - J\sum_{\langle i,j\rangle}(1+G_{ii}+G_{jj}-G_{ij}-G_{ji}) \times \exp\left[-(G_{ii}+G_{jj}-G_{ij}-G_{ji})\right]\cos(\phi_i-\phi_j). \tag{11}$$

Here matrices B_n and G are coupled by the equation

$$(\tilde{B}_n)_{ij} = \omega_n^2\left\{M_0 + \sum_{j'}\left[M_1 + \frac{\gamma}{2\pi\omega_n} + \frac{J}{\omega_n^2}\exp(-G_{ii}-G_{j'j'}+G_{ij'}+G_{j'i})\right]\right\}\delta_{ij} -\omega_n^2\left[M_1 + \frac{\gamma}{2\pi\omega_n} + \frac{J}{\omega_n^2}\exp(-G_{ii}-G_{jj}+G_{ij}+G_{ji})\right]\delta_{ij'}, \tag{12}$$

together with Eq. (10), where $M_0 \equiv C_0/4e^2$ and $M_1 \equiv C_1/4e^2$. Since we are interested in the low-temperature region, we have neglected thermal fluctuations in determining B_n and G; we further seek a uniform solution such that $G_{jj} \equiv G_0$ and $G_{jj'} \equiv G_1$, again reflecting the translational symmetry. With this, we obtain the equation for $g_0 \equiv \exp[-2(G_0 - G_1)]$ from Eqs. (10) and (12):

$$-\ln g_0 = \sum_{n=1}^{\infty}\frac{2}{\beta}\left[(\tilde{B}_n^{-1})_{ii} - (\tilde{B}_n^{-1})_{ii'}\right] = \sum_{n=1}^{\infty}\frac{2}{\beta M_0\omega_n^2}f(a_n),$$

where we have defined

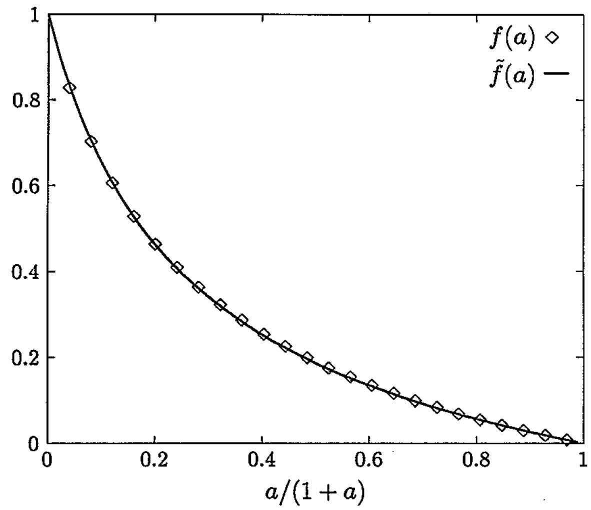

FIG. 1. $f(a)$ and $\tilde{f}(a)$ plotted as functions of $a/(1+a)$. The difference between them is small enough to be neglected.

$$f(a_n) \equiv \int_{-\pi}^{\pi} \frac{dk_x}{2\pi} \int_{-\pi}^{\pi} \frac{dk_y}{2\pi} \times \frac{1-\cos k_x}{1+4a_n - 2a_n(\cos k_x + \cos k_y)},$$

with $a_n \equiv M_1/M_0 + \gamma/2\pi M_0\omega_n + Jg_0/M_0\omega_n^2$. For simplicity, the function $f(a)$ may be replaced by

$$\tilde{f}(a) \equiv \frac{1}{4}\left[\frac{1}{1+2a} + \frac{1}{1/3+2a}\right],$$

which is an excellent approximation: Figure 1 shows the difference between $f(a)$ and $\tilde{f}(a)$, which is indeed small enough to be neglected.

The effective classical Hamiltonian (11) finally takes the form

$$H_{\rm cl} = -Jg\sum_{\langle i,j\rangle} \cos(\phi_i - \phi_j), \tag{13}$$

where $g = g(K,\alpha,\gamma,\eta)$ is determined by the equations

$$g = (1-\ln g_0)g_0\,,$$
$$-\ln g_0 = \frac{K\alpha}{8\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\left(1+2\eta+\frac{\alpha K\gamma}{2\pi^2 n}+\frac{K^2\alpha}{2\pi^2n^2}g_0\right)^{-1} + \left(\frac{1}{3}+2\eta+\frac{\alpha K\gamma}{2\pi^2 n}+\frac{K^2\alpha}{2\pi^2n^2}g_0\right)^{-1}\right], \tag{14}$$

with $K \equiv \beta J \equiv 1/T$. In Eq. (14), $\alpha \equiv 1/M_0J = 4e^2/C_0J$ measures the strength of the quantum fluctuations due to the self-capacitance and $\eta \equiv M_1/M_0 = C_1/C_0$ describes effects of the junction capacitance relative to the self-capacitance. In the self-charging limit ($\eta \to 0$), Eq. (14) reduces to the corresponding equation in Ref. 11. Equation (13) shows that dissipation and quantum fluctuations renormalize the coupling constant K to $Kg(K,\alpha,\gamma,\eta)$. At low temperature, for given α, γ, and η, there exist many roots of Eq. (14), among which the largest minimizes the effective classical Hamiltonian and thus should be chosen.

To study the critical behavior, we apply the RG technique to the effective classical Hamiltonian $H_{\rm cl}$ given by Eq. (13). Apart from the additional dependence of the renormalization factor g on η, the effective classical Hamiltonian is the same as that in the self-charging limit;[11] thus the scaling equations read

$$\frac{dK^{-1}}{dl} = \frac{g^2y^2}{g+K\partial g/\partial K},$$
$$\frac{dy}{dl} = (2-\pi Kg)y, \tag{15}$$

where y is the vortex fugacity. The phase diagram can be obtained by examining the behavior of the RG flow lines for given α, γ, and η, which is qualitatively similar to that in the self-charging limit:[11] In the absence of quantum fluctuations ($\alpha = 0$), the system becomes superconducting at temperatures below $T_c^{(0)}$ (≈ 0.95), regardless of the values of γ and η. For general α ($\neq 0$), the phase boundaries for given γ and η are thus determined by

$$T_c = T_c^{(0)} g(1/T_c, \alpha, \gamma, \eta).$$

Here the system at zero temperature ($T = 0$) needs particular attention since the summation in Eq. (14) does not converge. Instead it can be written in the integral form

$$\ln\left(\frac{8\pi^2}{\gamma^2\alpha}x^2\right) = \frac{x}{2\gamma}\int_0^\infty \left[\frac{1}{1+2\sqrt{xt}+(1+2\eta)t^2} + \frac{1}{1+2\sqrt{xt}+(1/3+2\eta)t^2}\right]dt$$
$$\equiv \frac{x}{2\gamma}h(x;\eta),$$

where $x \equiv \gamma(\alpha/8\pi^2 g_0)^{1/2}$ takes the place of g_0 and the integration variable is defined by $t = n(2\pi^2 g_0/\alpha K^2)^{1/2}$. The error in the replacement of the summation by the integral is of the order K^{-1}, which vanishes in the zero-temperature limit. The critical value of α at $T = 0$, denoted by α_1, then corresponds to the largest value of α, which yields a solution of x for given γ and η. For example, when $\gamma = 0$, α_1 is given by

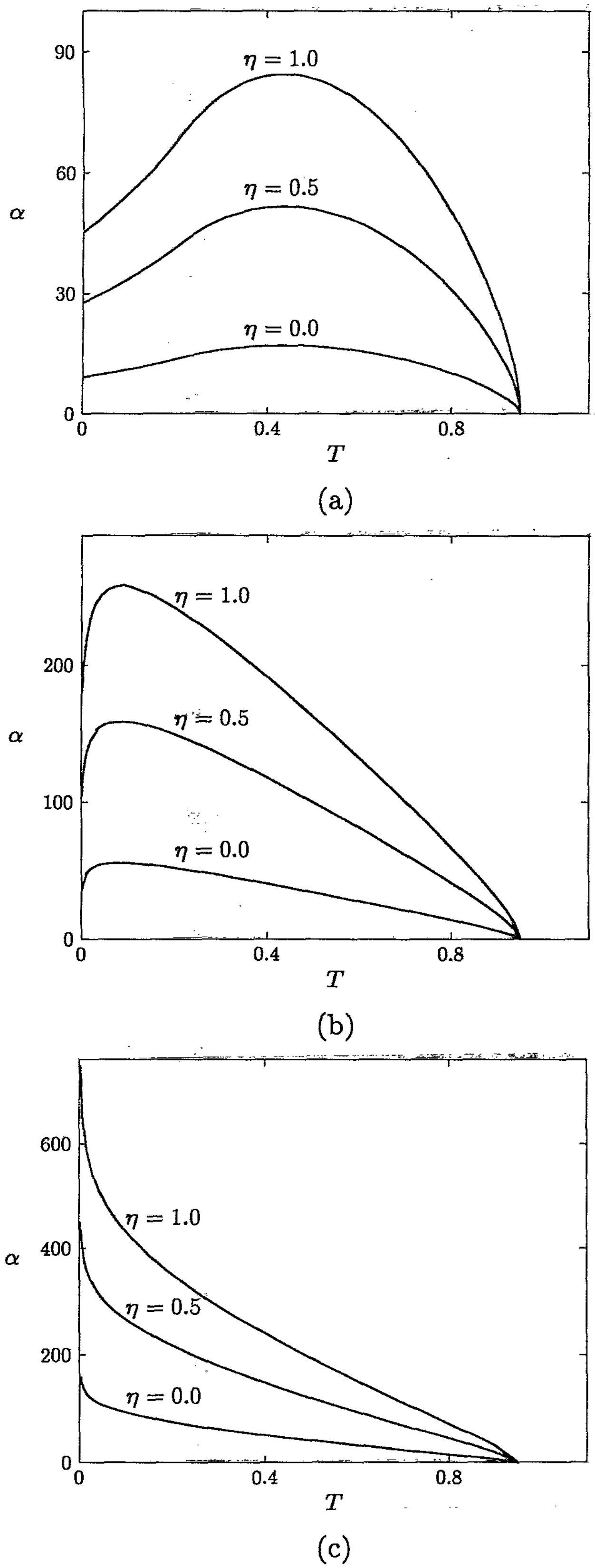

FIG. 2. Phase diagrams on the T-α plane for (a) $\gamma = 0.0$, (b) $\gamma = 0.4$, (c) $\gamma = 0.5$. The superconducting region expands with η, indicating the competition between C_1 and C_0. In (c), only the temperature range $T > 0.001$ is plotted: The reentrant behavior occurring at extremely low temperatures is not shown.

$$\alpha_1 = \frac{512}{\exp(2)} \left(\frac{1}{\sqrt{1+2\eta}} + \frac{1}{\sqrt{1/3+2\eta}} \right)^{-2}.$$

In the system quantum fluctuations associated with the self-capacitance tend to suppress superconductivity, thus lowering the critical temperature, while dissipation still tends to suppress quantum fluctuations. Interestingly, on the other hand, the junction capacitance competes with the self-capacitance, seemingly reducing some effects of the latter. Figure 2 shows the resulting phase boundary in the T-α plane for various values of γ and η. At weak dissipation there exist two characteristic values of α as functions of η: α_1 [$= 512\exp(-2)/(1+\sqrt{3})^2 \approx 9.3$ for $\gamma = \eta = 0$] dividing the normal and the superconducting states at zero temperature and α_2 (≈ 17.1 for $\gamma = \eta = 0$) beyond which no superconducting state is possible. For $\alpha < \alpha_1$, the system exhibits a KT transition into the superconducting state at a finite transition temperature T_c, which gets lower as α is increased. This KT transition persists for $\alpha_1 < \alpha < \alpha_2$; in addition, there exists a reentrant transition into the normal state at a lower transition temperature. As α is increased, these two transition temperatures approach each other, merging at $\alpha = \alpha_2$. For values of α beyond α_2, quantum fluctuations are strong enough to destroy superconductivity at all temperatures. Here it is of interest that the values of both α_1 and α_2 get larger as η is increased. Thus the junction capacitance competes with the self-capacitance and plays an interesting role in the transition of the system with *both* capacitances: While it tends to suppress the reduction of T_c, which results from the quantum fluctuations brought on by the self-capacitance, it also tends to lower the reentrance temperature. In this way, the junction capacitance apparently expands the range of reentrance as well as the region of superconductivity, by making the value of $(\alpha_2 - \alpha_1)$ larger, although larger values of α, i.e., stronger quantum fluctuations or smaller self-capacitance, are required to observe it. On the other hand, dissipation in the system tends to

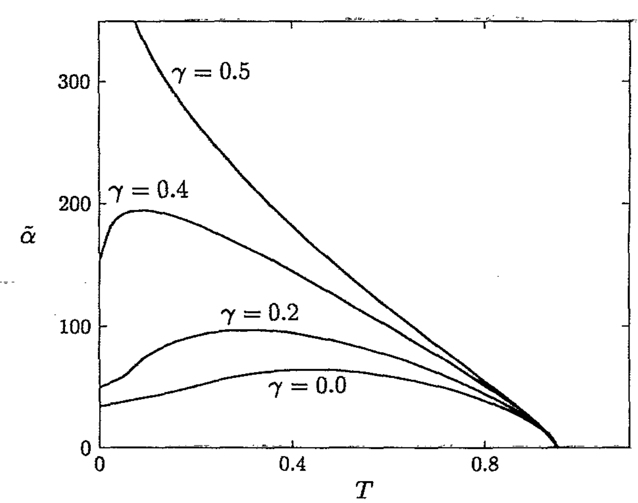

FIG. 3. Phase diagrams in the nearest-neighbor charging limit ($\eta = \infty$). Dissipation again suppresses reentrance.

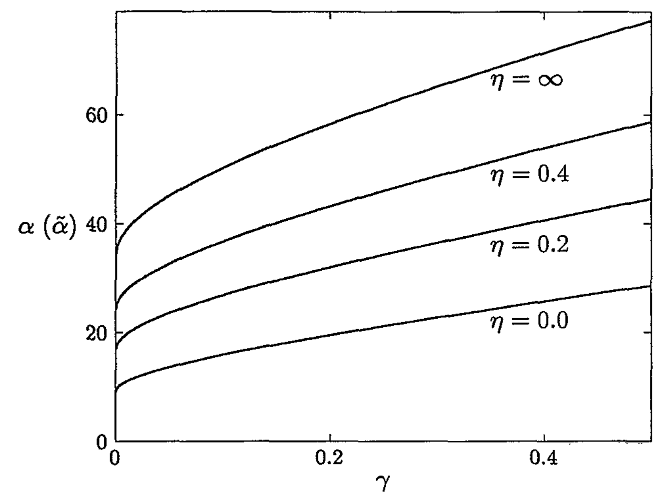

FIG. 4. Phase diagrams at zero temperature. The vertical axis denotes α for $\gamma < \infty$ and $\tilde{\alpha}$ for $\gamma = \infty$, respectively. Superconducting region expands as either γ or η is increased.

suppress quantum fluctuations in general, again expanding the superconducting region. The reentrant region, in contrast, shrinks rapidly as dissipation is increased: For example, the reentrant transition temperature at $\alpha = 50$ with $\eta = 0$ and $\gamma = 0.5$ has been calculated numerically and is found to be extremely low ($T_c < 1.0\times10^{-5}$), implying a vanishingly small reentrant region. Figure 3 shows the phase boundary of the system in the nearest-neighbor charging limit ($\eta = \infty$). In this case the phase boundary is drawn on the T-$\tilde{\alpha}$ plane, where $\tilde{\alpha} \equiv \alpha/\eta = 4e^2/C_1 J$ measures the strength of quantum fluctuations due to the junction capacitance. The phase diagram exhibits a structure largely similar to that of Fig. 2: There again exist two characteristic values $\tilde{\alpha}_1 [= 256 \exp(-2) \approx 34.6]$ and $\tilde{\alpha}_2 (\approx 64.1)$ such that for $\tilde{\alpha} < \tilde{\alpha}_1$, the system undergoes a KT transition at a finite temperature; in particular, for moderate fluctuations ($\tilde{\alpha}_1 < \tilde{\alpha} < \tilde{\alpha}_2$), there exists an additional low-temperature reentrant transition. However, note the large values of $\tilde{\alpha}$ in the reentrance region, compared with the corresponding values of α in Fig. 2. Figure 4 displays the phase boundary at $T = 0$ between the insulating phase and the superconducting one. For given γ and η, it shows that the system remains superconducting only when the amount α of quantum fluctuations does not exceed the critical value α_1. Comparing with Fig. 2(c), where the critical value of α at low but nonzero T is much larger than the corresponding value at $T = 0$ in Fig. 4, we conclude that, for strong dissipation, the reentrant behavior is confined to the region of extremely low temperatures.

It is well known that the allowed charge states of the system play an important role in the critical behavior. For example, in the self-charging limit, the coarse-graining approach yields reentrant behavior in the system of continuous charges, but not in the system of discrete charges.[6] It should be noted, however, that at $T = 0$ only the contributions from zero winding numbers survive in the path integral,[15] making the resulting phase diagram independent of the allowed charge states. Particularly, in the nearest-neighbor charging limit, the coarse-graining approach gives unphysical results as to the existence of the disordered phase for arbitrarily strong Josephson coupling.[6] This is to be contrasted with the duality between charges and vortices, which leads to the critical value $2C_1 J/e^2 = a\, 2/\pi^2$ between the insulating phase and the superconducting one. Here a is a constant of value slightly larger than unity, implying that the duality is not exact.[15] The critical value in this paper, $\tilde{\alpha}_1 \approx 34.6$, corresponds to $a \approx 1.14$, which is in good agreement with the expected value.

We next consider the array in the presence of a uniform external magnetic field, which is described by the Hamiltonian

$$H = \frac{1}{2}\sum_{i,j}(q_i - q_0)C_{ij}^{-1}(q_j - q_0) - J\sum_{\langle i,j\rangle}\cos(\phi_i - \phi_j - A_{ij}), \qquad (16)$$

where A_{ij} is the line integral of the vector potential $\mathbf{A}$:

$$A_{ij} = \frac{2e}{\hbar c}\int_i^j \mathbf{A}\cdot d\mathbf{l}.$$

The sum of A_{ij} around a plaquette then becomes $2\pi f$, where the frustration f is given by the magnetic flux per plaquette in units of the flux quantum $\Phi_0 (\equiv hc/2e)$. Here we concentrate on the fully frustrated system ($f = 1/2$), the ground state of which is doubly degenerate:[16]

$$|\phi_i - \phi_j - A_{ij}| = \pi/4 \ (\mathrm{mod} 2\pi).$$

The remaining procedure is entirely similar to the previous one used for the unfrustrated system; it leads to the effective Hamiltonian valid at low temperatures,

$$H_{\mathrm{cl}} = -Jg\sum_{\langle i,j\rangle}\cos(\phi_i - \phi_j - A_{ij}), \qquad (17)$$

which describes the frustrated XY model.[1] Here g represents the renormalization of the coupling by quantum fluctuations, and is given by the largest root of the equations

$$\begin{aligned} g &= (1 - \ln g_0) g_0, \\ -\ln g_0 &= \frac{K\alpha}{8\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} \\ &\quad\times\left[\left(1 + 2\eta + \frac{\alpha K\gamma}{2\pi^2 n} + \frac{K^2\alpha}{2\sqrt{2}\pi^2 n^2} g_0\right)^{-1} \right. \\ &\quad \left. + \left(\frac{1}{3} + 2\eta + \frac{\alpha K\gamma}{2\pi^2 n} + \frac{K^2\alpha}{2\sqrt{2}\pi^2 n^2} g_0\right)^{-1}\right]. \qquad (18) \end{aligned}$$

The usual (classical) frustrated XY model ($g = 1$) has been studied quite extensively.[1] The fully frustrated system is well known to possess both a continuous U(1) symmetry and a discrete Z_2 symmetry associated with the double degeneracy, thus giving the possibility of an Ising-type transition (for chirality) in addition to the un-

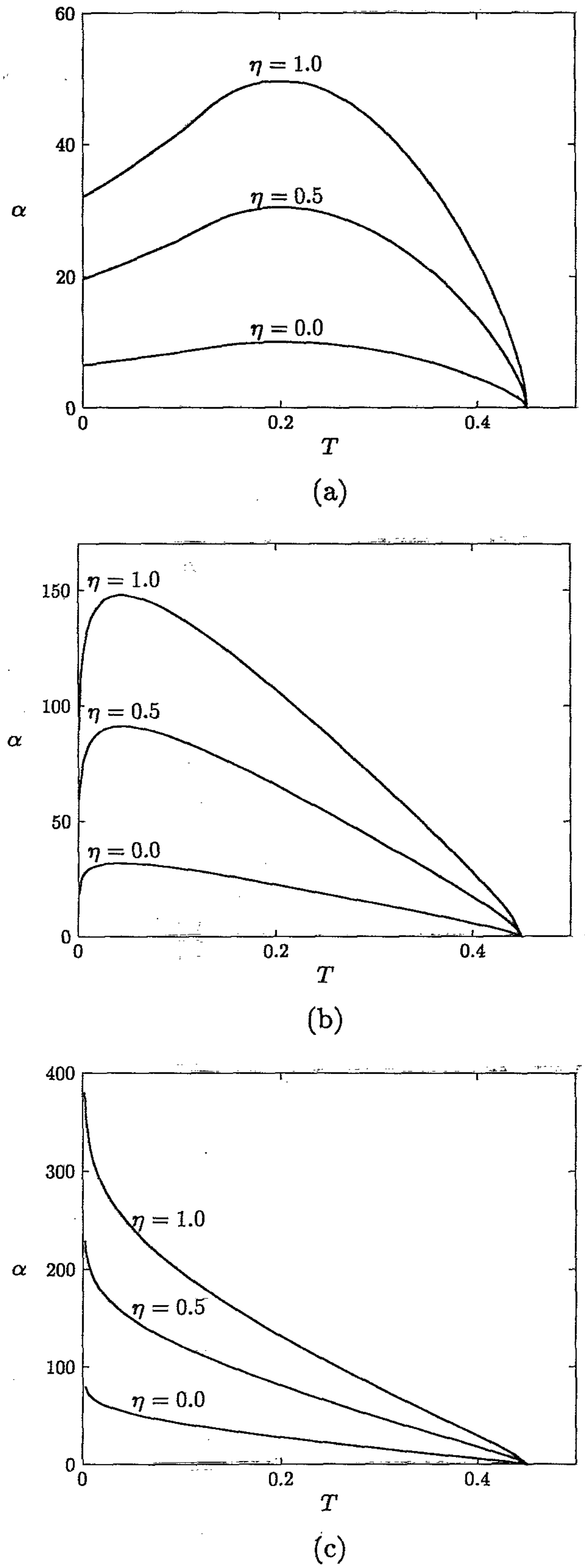

FIG. 5. Phase diagrams in the presence of an external magnetic field ($f = 1/2$), for (a) $\gamma = 0.0$, (b) $\gamma = 0.4$, (c) $\gamma = 0.5$. *Only* the temperature range $T > 0.001$ is plotted in (c). The general features are qualitatively similar to those of Fig. 2.

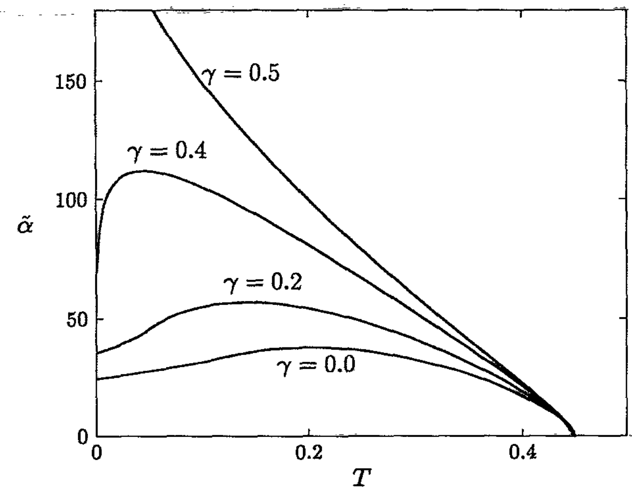

FIG. 6. Phase diagrams for $\eta = \infty$ in the presence of an external magnetic field ($f = 1/2$).

derlying KT-type one (for phases).[17] In spite of a number of studies of the fully frustrated system, the precise nature of the overall transition is still somewhat unclear: In particular, there has been controversy as to whether the two transitions occur simultaneously at the same temperature.[18,19] In this paper we consider only two states in the fully frustrated system: the low-temperature superconducting states, where there exists long-range order for chirality as well as algebraic order for phases, and the high-temperature normal state, where domain walls and vortices destroy such long-range order and algebraic order, respectively.[17] Figures 5 and 6 display the corresponding phase boundaries for various values of γ and for $\eta \leq 1$ and $\eta = \infty$, respectively. They show that the overall effects of quantum fluctuations on the fully frustrated system are qualitatively similar to those on the unfrustrated system investigated earlier: At weak dissipation, moderate quantum fluctuations allow a low-temperature reentrant transition regardless of the value of η although the junction capacitance in general suppresses the decrease of the superconducting transition temperature due to the self-capacitance and lowers the reentrant transition temperature. Here, however, additional topological excitations associated with domain walls result in smaller values of α_1 [$= 512\exp(-2)/\sqrt{2}(1+\sqrt{3})^2 \approx 6.56$ for $\gamma = \eta = 0$], α_2 (≈ 10.1 for $\gamma = \eta = 0$), $\tilde{\alpha}_1$ [$= 256\exp(-2)/\sqrt{2} \approx 24.5$ for $\gamma = 0$, $\eta = \infty$], and $\tilde{\alpha}_2$ (≈ 37.7 for $\gamma = 0$, $\eta = \infty$), leading to a range of reentrance narrower than that of the unfrustrated system. In the particular case of the self-charging limit ($\eta = 0$) without dissipation ($\gamma = 0$), the coarse-graining approach was also applied both to the unfrustrated system ($f = 0$) (Ref. 6) and to the fully frustrated system ($f = 1/2$),[20] yielding the critical values $\alpha_1(f = 0) = 8$ and $\alpha_1(f = 1/2) = 4\sqrt{2}$. It is of interest that the corresponding values obtained in this paper give the same ratio $\alpha_1(f = 0)/\alpha_1(f = 1/2) = \sqrt{2}$ although each value is somewhat different.

In conclusion, we have studied the effects of quantum fluctuations together with dissipation on the two-dimensional superconducting arrays having both the self-capacitances and junction capacitances. We have used the rather accurate variational method in computing the Feynman path integral, and obtained the effective classical Hamiltonian with the coupling renormalized by quantum fluctuations. The renormalization-group technique has then revealed a general suppression of superconductivity by quantum fluctuations and the existence of a low-temperature reentrant transition for moderate quantum fluctuations. Here dissipation suppresses quantum fluctuations in general, tending to restore superconductivity: In particular strong dissipation shrinks the reentrant region strikingly and restores superconductivity at low temperatures, even for large quantum fluctuations. The junction capacitance is also found to suppress some effects of the quantum fluctuations induced by the self-capacitance: It tends to restrain the reduction of the superconducting transition temperature and to expand the superconducting region, similarly to dissipation. In contrast with dissipation, however, the junction capacitance does not suppress reentrance; rather, it appears to increase the range of reentrance although we need large quantum fluctuations for the observation of reentrance in such an array with non-negligible junction capacitance. In the absence of the self-capacitance ($\eta = \infty$), the junction capacitance alone still introduces quantum fluctuations, yielding reentrance for appropriate values of the parameters. If only discrete charge states are allowed in this nearest-neighbor charging limit, which describes the ideal system without (external) dissipation, there exists the charge-vortex duality, giving interesting implications.[15] On the other hand, the continuous charge variables considered in this paper do not allow the duality between charges and vortices, and the charging-energy term only renormalizes the coupling strength. Nevertheless, unlike the coarse-graining approach, the phase boundaries at zero temperature, where the distinction between the two charge states disappears, indeed exhibit results consistent with those from the duality argument. The presence of frustration due to an external magnetic field has been also considered with regard to the effects of dissipation and quantum fluctuations, which results in qualitatively similar features.

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